TREATMENT OF FIELD SINGULARITIES IN THE FINITE-DIFFERENCE APPROXIMATION

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ABSTRACT

Due to the discretization of space, the Finite-Difference method cannot resolve exactly singularities of the electric and magnetic field at metallic edges. This paper presents a simple method which reduces the corresponding error without increasing the numerical efforts. The new edge formulation is tested by calculating the line parameters and the field pattern of an homogeneous MMIC coplanar-waveguide.

MOTIVATION

Purely numerical field-theoretical methods such as the Finite-Difference, the Finite-Element, or similar approximations have the advantage that one can handle nearly arbitrarily shaped structures. On the other hand, areas of strong local changes in the electromagnetic field like singularities at metallic edges are resolved with poor accuracy only unless...

> ...an appropriate number of mesh lines is chosen in this region which, however, often leads to excessive computational efforts.
>
> ...local mesh refinement is used which enables one to discretize only the region of interest by a finer mesh [1].
>
> ...the known field behavior is incorporated in the discretized Maxwell equations, which does not increase numerical efforts [2,3,4].

Beyond that, empirical approaches have been developed to treat the problem of field singularities at metallic edges for special cases.

But most of the methodes based on the incorporation of the known field behavior as well as those using empirical approaches can deal only with special geometries, particularly the edges of infinitely thin metallic strips. Compared to the general case, this represents a significant simplification because electric and magnetic fields here exhibit the same order of singularity, namely $r^{-1/2}$.

The influence of the error on the overall electromagnetic behavior depends on how significant the singularity is for the total pattern. For example, considering a microstrip the edge effect is less pronounced than in the coplanar waveguide (CPW) case, because the CPW electric and magnetic fields are concentrated in the slot bounded by two edges. Therefore, analysing a CPW by the Finite-Difference (FD) method with a reasonably sized grid one finds considerable errors, especially with regard to the characteristic line impedance Z_c. Our calculations [8], for instance, lead to about 10-20% deviation in Z_c compared to an analytical formula.

Hence, an improved formulation is required which is applicable to an arbitrary geometry without increasing the numerical efforts. In the following, such a method is presented which also includes different orders of singularities for magnetic and electric field at the same edge which occur due to different material properties ε_r and μ_r. The validity of the new edge formulation is checked by calculating the characteristic line parameters of an homogeneous coplanar line with MMIC typical dimensions.

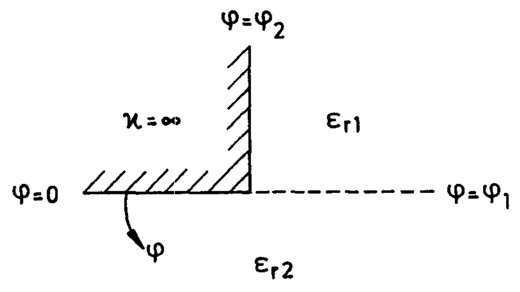

Fig. 1: Cross-section of a metallic edge ($\varphi_2 < \varphi < 2\pi$) with two dielectrics ($\varepsilon_{r1} \rightarrow \varphi_1 < \varphi < \varphi_2$; $\varepsilon_{r2} \rightarrow 0 < \varphi < \varphi_1$).

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The description given here is limited to the case of a metallic edge embedded in a non-homogeneous dielectric material. This method, however, can also be applied to other geometries such as dielectric edges.

METHOD

As well known [5], the electric and magnetic fields at metallic edges show a slightly different order of singularity if more than one dielectric material is involved ($\vec{E} \sim r^{\nu_E - 1}$, $\vec{H} \sim r^{\nu_H - 1}$). In Fig. 1, an example is given that corresponds to the situation at the lower edge of the metallization of a coplanar waveguide.

Employing the Finite-Difference, the whole space is discretized by a non-equidistant cartesian grid. All material boundaries are located on mesh lines. The electric and magnetic field quantities are defined according to Yee's grid [6].

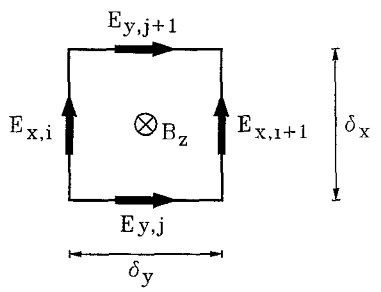


Fig. 2: The Finite-Difference mesh with field quantities and dimensions.

For the implementation of the edge formulation we consider the second Maxwellian equation in integral form:

$$\oint_C \vec{\mathbf{E}}\, d\vec{s} = \int_A (-j\omega \vec{\mathbf{B}}) d\vec{A} \qquad (1)$$

Applying the Finite-Difference scheme [7,8] to a grid as illustrated in Fig. 2 converts the integral along a mesh line according to the following equation:

$$\int_0^{\delta_x} E_x(x)\, dx \overset{\text{FD}}{\approx} \int_0^{\delta_x} E_x(\delta_x/2)\, dx = E_{x,i}\, \delta_x \qquad (2)$$

Thus, the FD formulation of eqn (1) reads:

$$E_{x,i}\, \delta_x - E_{y,j}\, \delta_y - E_{x,i+1}\, \delta_x + E_{y,j+1}\, \delta_y = -j\omega B_{z,k} \delta_x \delta_y \quad (3)$$

If a grid point is located directly at a metallic edge the electric field behavior along the mesh line perpendicular to the conducting surface can be assumed to follow an $r^{\nu-1}$ law, where $\nu - 1$ denotes the order of the singularity. Incorporating this knowledge into the FD scheme eqn. (2) reads:

$$\int_0^{\delta_x} E_x(x)\, dx \overset{\text{FD}}{\approx} \int_0^{\delta_x} E_x(\delta_x/2) \left(\frac{\delta_x/2}{x}\right)^{1-\nu} dx = E_{x,i}\, \delta_x\, K_E \qquad (4)$$

$$\text{with} \qquad K_E = \frac{1}{\nu \cdot 2^{1-\nu}}$$

It is interesting to note that the correction factor K_E depends solely on the order of the field singularity ν and not on the grid size.

Corresponding to the derivation of K_E one can obtain also a correction factor K_H for the magnetic field. One should state that the general numerical properties of the matrix system to be solved are not changed if K_E and K_H are considered in the discretized Maxwell equations (eqn. 3).

The main advantage of this new edge formulation is that it is based on an analytical derivation. Therefore, it can be extended to all types of edges where field singularities can occur. It is even possible to consider three-dimensional corners provided the order of singularity is known a priori.

RESULTS

In order to show how the improved formulation behaves, an MMIC coplanar line (see Fig. 3) is investigated.

All boundary conditions of the CPW considered are located sufficiently away from the slots to minimize their influence. To check the validity, results from a conformal mapping technique [9] are included which can consider finite metallization thickness. The error of this analytically based method remains smaller than 1% for the structures considered. The frequency of operation in the Finite-Difference method is set to 1GHz to avoid dispersion due to non-TEM effects.

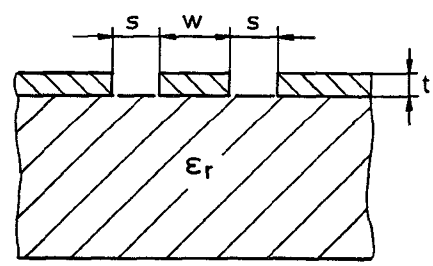


Fig. 3: Coplanar waveguide ($w = 15\mu m$, $s = 10\mu m$, $t = 0$ and $3\mu m$, $\varepsilon_r = 12.9$).

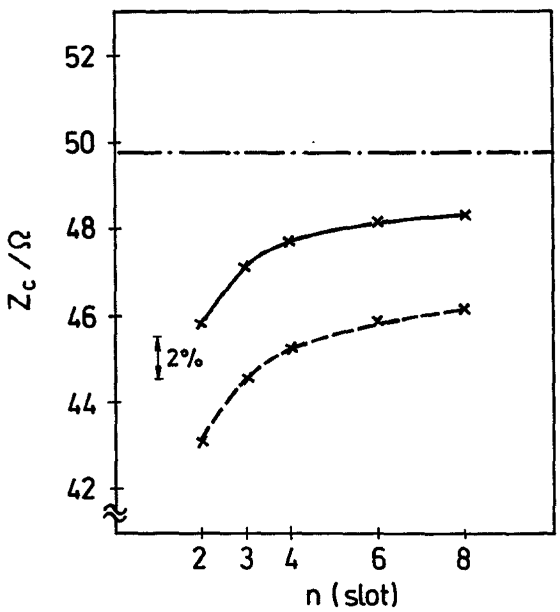


Fig. 4: Characteristic line impedance Z_c of a CPW ($t = 0$) against number of mesh points in the slot.
—— : FD-method incorporating the edge condition
- - - : conventional FD-method
- · - : conformal mapping method

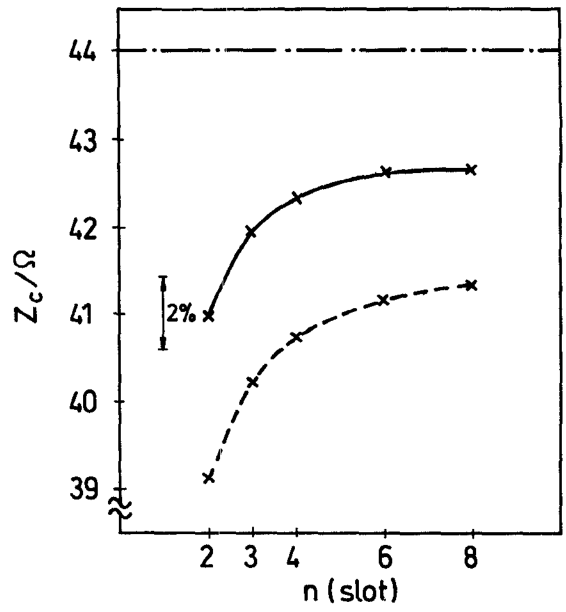


Fig. 5: Characteristic line impedance Z_c of a CPW ($t = 3\mu m$) against number of mesh points in the slot.
—— : FD-method incorporating the edge condition
- - - : conventional FD-method
- · - : conformal mapping method

The curves presented in the following are plotted as a function of the number of grid points in the slot region. Mesh lines in other regions are not varied to obtain comparable results. The whole structure is discretized by 20 and 22 steps in vertical direction for $t = 0\mu m$ and $t = 3\mu m$, respectively, and by $19 + n(slot)$ in horizontal direction.

The characteristic line impedance Z_c of the CPW is presented in Fig. 4 for zero metallization thickness. Incorporating the edge condition reduces the deviations to about one half. The remaining error can be mainly contributed to the discretization itself which is still only an approximation of the real electromagnetic field. For the special structure with $t = 0$, the propagation constant k_z does not deviate from the analytical value.

In Figs. 5 and 6, Z_c and k_z/k_0 with $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, respectively, are plotted against the number of mesh points in the slot region for a CPW with a metallization thickness of $t = 3\mu m$. The error of the characteristic line impedance is again reduced by about one half using the new formulation. In contrast to the structure with $t = 0$, also the propagation constant shows a deviation of about 2% between the conformal mapping and the conventional Finite-Difference method, which is reduced significantly by the improved formulation.

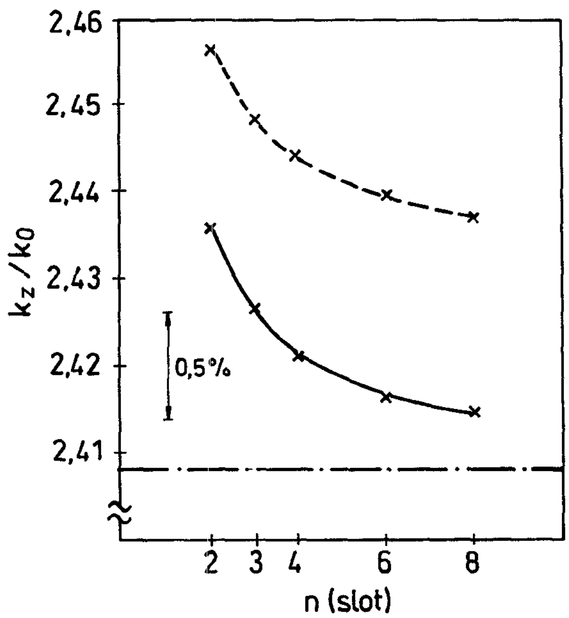
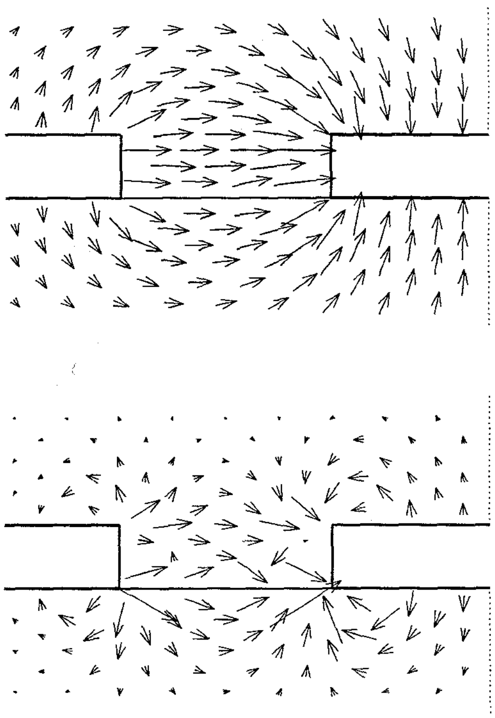


Fig. 6: Normalized propagation constant k_z/k_0 of a CPW ($t = 3\mu m$) against number of mesh points in the slot ($k_0 = \omega\sqrt{\mu_0\varepsilon_0}$).
—— : FD-method incorporating the edge condition
- - - : conventional FD-method
- · - : conformal mapping method

Fig. 7 illustrates the differences in the calculated electric field between the two FD formulations. Both the field pattern obtained by the improved Finite-Difference approximation and the vector-field difference are plotted. The results verify that primarily the field quantities at the edges are influenced by the new edge formulation.

One should emphasize that under quasi-TEM conditions, the magnetic field symmetry of the upper and the lower half plane must be symmetrical. This property is maintained only if different singularity factors for the electric and magnetic fields are considered.



a)

b)

Fig. 7: Electric field pattern in the slot region (n(slot) = 4):
a) FD-method with improved formulation
b) Difference between conventional FD and case a) magnified by a factor of 10.

CONCLUSIONS

- Numerical field-theoretical methods such as Finite-Difference or Finite-Element yield errors due to poor resolution of field singularities at metallic edges. For the CPW, for instance, this results in typical deviations of 10-20% in the characteristic line impedance.
- An improved formulation is developed by incorporating a correction factor depending on the field singularity at the edge. It reduces the errors significantly by a factor of about one half.
- The concept can be extended to treat also the case of three-dimensional corners.

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